Getting Creative with Systems of Equations

A Cinematic Lesson

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Overview

- Cinematic Lesson Structure
- One-Page Lesson Plan
- Sample Lesson – *Selecting Systems of Linear Equations*
- Debrief
- Q&A


Inciting Incident


Many
BRAVE KNIGHTS
had attempted to
FREE HER FROM THIS
DREADFUL PRISON
BUT NONE PREVAILED


Inciting Incident

Pixar Animation Studios

- *Toy Story* – favorite toy replaced by new model
- *Monsters, Inc* – monsters are deathly afraid of kids but one sneaks into their world
- *Finding Nemo* – overprotective father loses his son
- *Wall-E* – a lone robot on desolate Earth is suddenly visited by a beautiful new robot from outer space
- *Inside Out* – fun and happy adolescent girl has to move

Inciting Incidents in Film & Literature

- Major event that propels action and characters forward, setting the story into motion.
- Should…
  - Happen within the first 25% of the story
  - Directly involve the protagonist (or require them to act/respond)
  - Set the stage for revelation or growth for the protagonist
  - Hook the reader and spur questions

Some Types of Inciting Incidents

- Leaving home (“Hero’s Journey”)
- Life interrupted (a big change to self or circumstances)
- Scary discovery (or learning a dark secret)
- I need to help (but there will be personal cost)
- Joining a coveted group or institution
- Whodunnit?
- Howcatchem?

How can we use
inciting incidents
in math lessons?

Introduction to Quadratic Functions (Non-inciting)

Introduction to Quadratic Functions (Non-inciting)

Quadratic equations are second-order polynomials, and have the form $y = ax^2 + bx + c$

Their graphs make a shape called a parabola.

Introduction to Quadratic Functions (Inciting)

Introduction to Quadratic Functions (Inciting)

$f(x) = x^2$

$y = 3x^2 - 5x + 7$

$g(x) = 10x^2 + 27$

$m = n^2 + 4n$

$f(x) = |x^2 - 4|$

$y = 0.7x^2 + x + \frac{1}{2}$

$h(x) = 24 - x^2$

$y = 2x + 3$

$x^2 + 1 = y$

$f(x) = 2(x - 3)^2$

$g(x) = x^3 - x^2 + x + 4$

$f(x) = x^2 + \frac{1}{x}$

$y = 2$

$h(x) = x^4 + 2x^3 + 2x + 1$

Introduction to Quadratic Functions

QUADRATIC FUNCTIONS

$$m = n^2 + 4n$$

$$f(x) = x^2$$

$$g(x) = 10x^2 + 27$$

$$y = 0.7x^2 + x + \frac{1}{2}$$

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$$h(x) = 24 - x^2$$

NOT QUADRATIC FUNCTIONS

$$f(x) = |x^2 - 4|$$

$$y = 2x + 3$$

$$g(x) = x^3 - x^2 + x + 4$$

$$h(x) = x^4 + 2x^3 + 2x + 1$$

$$y = 2$$

$$f(x) = x^2 + \frac{1}{x}$$

Graphing a Polynomial (Non-inciting)

Graphing a Polynomial (Non-inciting)

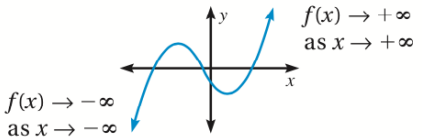
A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

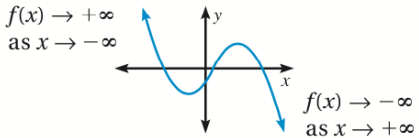
where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. For this function, a_n is the **leading coefficient**, n is the **degree**, and a_0 is the **constant term**. A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

End Behavior of Polynomial Functions

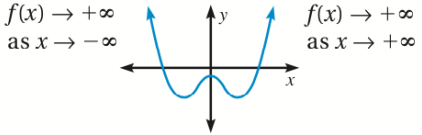
Degree: odd
Leading coefficient: positive



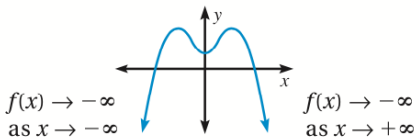
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive

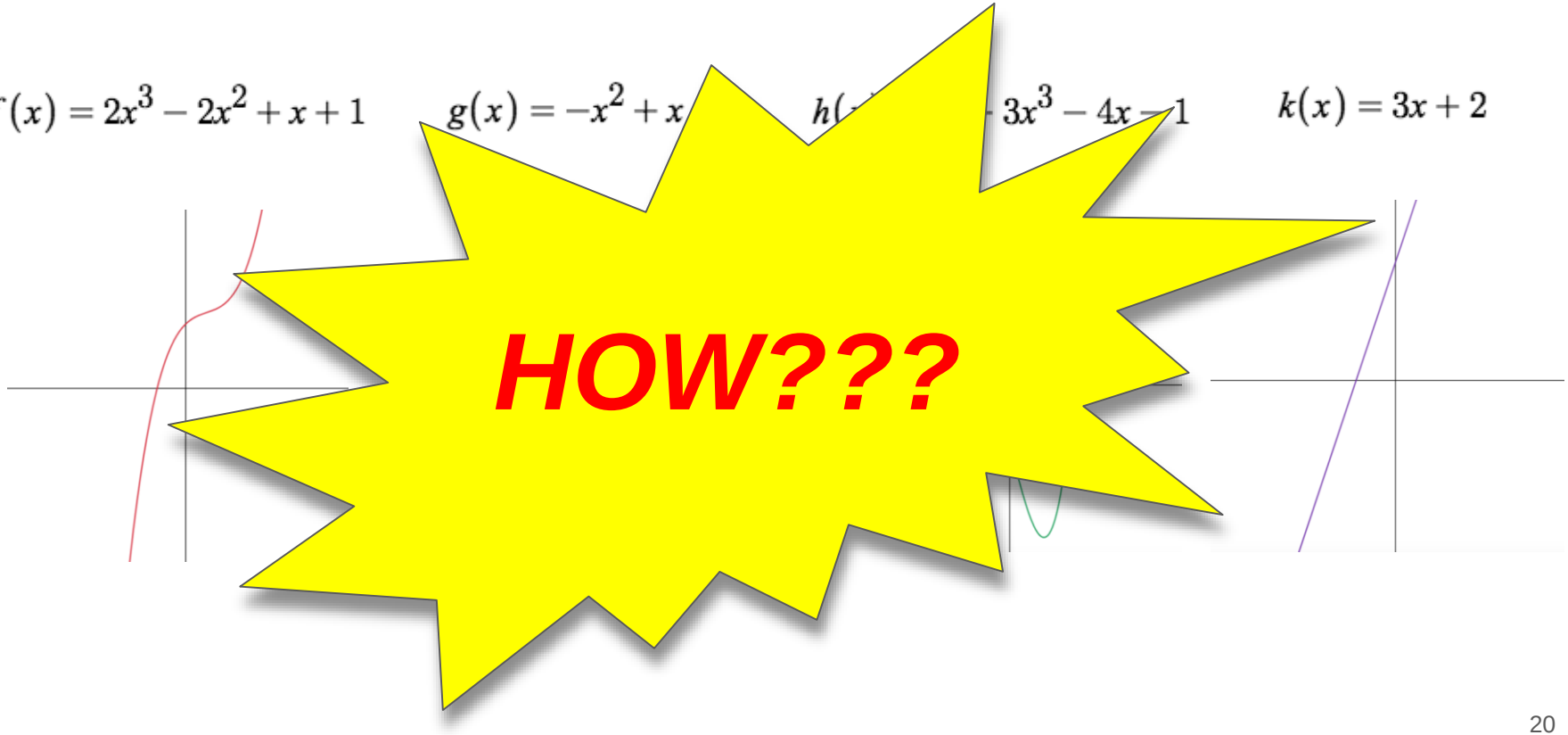


Degree: even
Leading coefficient: negative



Graphing a Polynomial (Inciting)

$f(x) = 2x^3 - 2x^2 + x + 1$ $g(x) = -x^2 + x$ $h(x) = 3x^3 - 4x - 1$ $k(x) = 3x + 2$



Some Types of Inciting Incidents in Math Lessons

- Guess the rule (e.g., quadratics vs. non-quadratics)
- Gain an insight (e.g., polynomial graphs)
- Throw a wrench in it (e.g., systems of equations, *coming soon*)
- Can we extend it?
- Can we predict it?
- Does it always work?
- Are they the same?

Examples of these? Or others categories?

Inciting Incidents in Math Lessons

RECALL

- Major event that propels action and characters forward, setting the story into motion.
- Should…
  - Happen within the first 25% of the story
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Inciting Incidents in Math Lessons

Foster curiosity

Focus on mathematical ideas (*the protagonist!*)

Provide a rationale and motivation for the lesson

Is a good inciting incident just an inciting incident, or does it influence the remainder of the lesson?

Thinking about Full Lesson Plans

Cinematic Storytelling Structure

- Setup
- Inciting Incident
- Rising Action
- Climax
- Resolution (what was learned, new normal, sequel?)

- “Show, don’t tell.”

Snyder “Save the Cat”; Dietiker (2013)

Cinematic Structure

- *Rear Window* – a homebound photograph snoops on his neighbors and notices what might be a domestic murder
- *Shrek* – a reclusive ogre finds his swamp invaded
- *Toy Story* – favorite toy replaced by new model
- *Finding Nemo* – overprotective father loses his son
- *Wall-E* – a lone robot on desolate Earth is suddenly visited by a beautiful new robot from outer space
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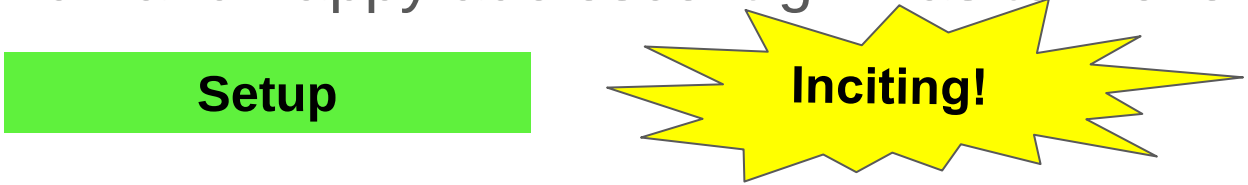

Setups

- Setting
- Main character (*mathematical object or phenomenon*)
- Something to be learned

In service of the inciting incident.

Some setups can be very short, some a bit more involved.

Remember 25% guideline.

Setups

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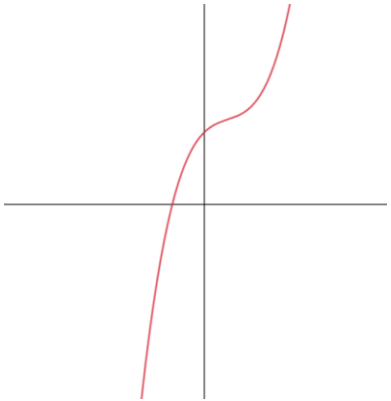
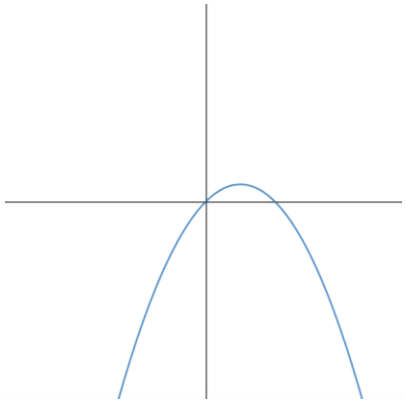
Setups

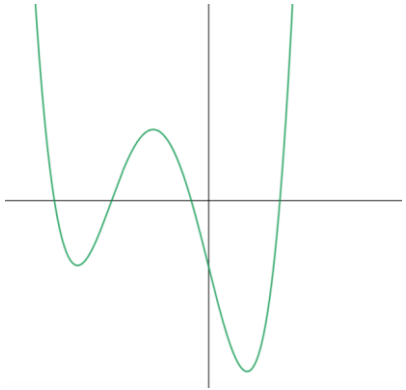
$f(x) = 2x^3 - 2x^2 + x + 1$



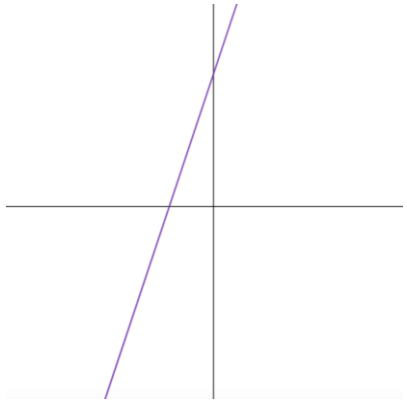
$g(x) = -x^2 + x$



$h(x) = x^4 + 3x^3 - 4x - 1$



$k(x) = 3x + 2$



One-Page Lesson Plan

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https://tinyurl.com/1pagesystems

TITLE

Goal(s)

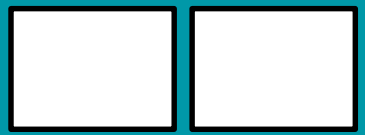
Standard(s)

Setup

Inciting Incident

Rising Action

Climax



Resolution

Selecting Systems of Equations

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Otten & Otten (2017)

https://tinyurl.com/1pagesystems

Systems of Equations

Solution Methods

- Substitution
- Elimination/Combination
- Graphing
- Inspection

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Systems of Equations

Thus far, you've been given systems to solve.

$$\begin{cases} 2x-3y=0 \\ -4x+2y=-8 \end{cases}$$

$$\begin{cases} y=\frac{1}{2}x+2 \\ -2y=-x-4 \end{cases}$$

$$\begin{cases} 4x+2y=9 \\ 3x-2y=10 \end{cases}$$

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Systems of Equations

Today you get linear equations.

A) $5x - 2y = 8$

B) $2x = 12$

C) $-2x + 4y = 12$

D) $y = -3$

E) $4y = 2x$

F) $y = -\frac{x}{2} + 2$

G) $3x + 2y = 0$

H) $9x + 6y = 0$

What sort of variety do you see in these equations?

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Systems of Equations

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What would you look for if **you got to pick** which equations go together into a system?

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C) $-2x + 4y = 12$

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H) $9x + 6y = 0$

- Pick 2 equations for **substitution** and solve
- Pick 2 equations for **elimination/combination** and solve
- Pick 2 equations for **graphing** and solve
- You can only use an equation once, so be strategic!

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What did you look for when choosing?

Solution Methods

- Substitution – *solved for a variable; same term in two equations*
- Elimination/Combination – *multiple of one term appears in another equation (they cancel); standard form preferred but not required*
- Graphing – *equations easy to graph (slope-intercept form, horizontal or vertical)*

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- Is there a pair that you could solve in your head?
- Do any pairs of lines have *no solution*? How do you know?
- Is it possible to have a solution to a system with *three equations*?

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Exit Slip

Which 2 equations would you choose for your system if you had to solve it by the **substitution method**? Why would you choose those 2? *Note: You don't have to solve the system.*

A. $y + 5x = 2$

C. $-7x + 3y = 3$

B. $5x = 2y + 11$

Lesson Debrief

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Lesson Debrief

- “Throw a wrench in it” – pick equations instead of pick the solution method
- Forces conversation to be about the structure of the equations and flexibility in solution processes, not about steps to follow or answers.
- No single “right” answer, but there are more or less strategic ones.

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Overall thoughts? Questions?

Q&A

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