ACTIVITY 3 Exploring Visual Patterns—A Scaffolded Approach

MATERIALS:

- The Arch Problem, part 1 (handout)
- The Arch Problem, part 2 (handout)
- The Arch Problem, part 3 (handout)
- Different color markers

STEPS:

1. Write, **Algebra is the generalization of arithmetic** on the board. Ask students what *arithmetic* is. Then ask them what a *generalization* is. Ask students what they think the sentence means.

2. Tell students: *In order to help them learn algebra, I want to start working from what you already know. You are going to look at a visual pattern and collect information on how it is changing. Then we are going to work together to use algebra to generalize the patterns we find.*

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> This line of discussion is adapted from *A Collection of Math Lessons, Grades 6–8* by Marilyn Burns and Cathy Humphreys.

3. Hand out The Arch Problem, part 1, and give students two minutes to look at the pattern and write a few observations down. Then have them talk in pairs about what they see in the Arch Problem. While they are talking, draw the first three figures on the board, large enough to be seen in the back of the room.

4. Once your students have had some time to make some observations, bring the class back together and ask them to share the changes they notice in each figure.

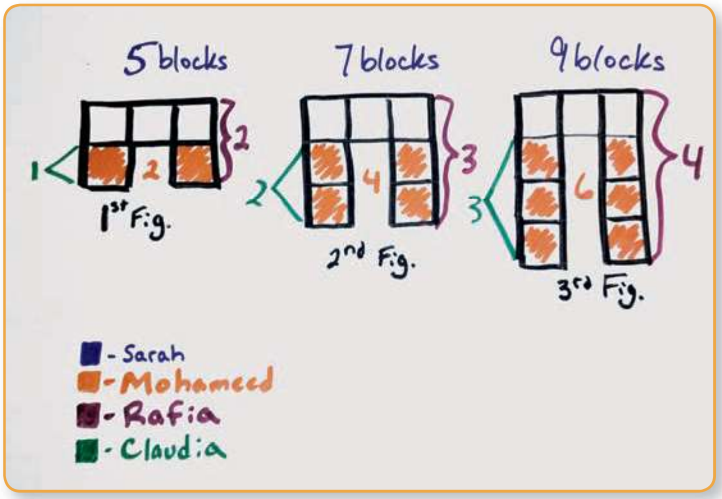
   Some things they might say:

   "The number of squares goes up by 2."

   "The number of squares underneath the top row goes up by 2."

   "As the figure number changes, the figure gets taller by 1."

5. As each student shares one of their observations, stop and ask them to explain so everyone can see it. If, for example, a student says, *"The number of squares goes up by two,"* ask her to point out specifically where she sees that in the drawing. Label the observation on the picture you drew on the board with a color and write the student's name below, in the same color. After you have recorded a few student observations, the picture might look something like this:



Identifying each student's contribution is helpful because as you go through the next activity, it allows everyone to talk about certain patterns by name—i.e., "Rafia's pattern"—instead of having to describe the whole pattern each time someone wants to reference it. Also, since everyone's heads are full of patterns they want to share, it's important to make sure everyone is listening to each specific pattern being shared.

6 Once all of the observations have been shared, tell the class they are going to use their collective observations to make predictions. Have your class get into groups of 2-3 students and hand out ARCH PROBLEM, PART 2.

7 Walk around the room as the groups are working. Listen for interesting discussions, disagreements and struggles to raise during the whole class discussion. As you walk around, keep in mind the things you want to come out of the discussion for each question (detailed below) and look for connections.

Some things you should listen for/ask about:

a. When students sketch the next two figures, do their figures follow all of the patterns identified and shared in step 4?

b. How do students start their sketch of the next two figures? Some might draw three squares across and then draw the "legs" coming down. Some might draw one square in the middle and then draw the legs coming down. Some might draw the first figure and then add some squares to the bottom of each "leg." Try to take note of who is doing what.

c. Instead of describing what the tenth figure will look like, some groups might write, "The 10th figure has 23 squares in it." If you come across this, ask them to read the question again.

7a

When beginning to work with visual patterns, there are often 1-2 students who struggle with drawing the next two figures. It depends on which observation(s) they are using. Say for example they are only focusing on the number of blocks. There are 11 squares in the 4th figure. Eleven squares can be arranged in a lot of different ways. You can help these students by referring them to all of the patterns identified in steps 3 and 4. It not only has to have 11 squares, but it has to have a height of 5, etc.

7b

The first two ways are very common and almost always both come up. The third way is less common, and needn't be forced, but is a special treat when a student see the figures in that way.

Ask them, *"What will those 23 squares look like? How will they be arranged?"*

d. Some students might assume that if the 5th figure has 13 squares, the 10th figure will have double that amount, or 26 squares. If that happens, you might ask if there are any patterns they notice in the table. They probably already noticed that the number of squares goes up by 2. Ask them to continue the table and see if they get the same answer. Then walk around some more—give the group a chance to discuss on its own which answer they think is correct—the 23 or the 26. Check back in a few minutes and see where their conversation is. If it has stalled and there is no consensus, you can them if the 4th figure has twice as many squares as the 2nd figure, but it is much better if they can get there on their own. If they do, make sure to ask how they decided.

e. Which group has a clear set of steps for figuring out how many squares are in the 99th figure? Ask them to talk you through their steps. Look for one or two that are clear and ask the group if they would be willing to share their method during the debrief. Ask them to spend a few minutes talking about how they will demonstrate their method so it is clear to the other folks in the class.

8 Once the groups are mostly done with the first 5 questions (don't worry if no one has gotten to the bonus question), bring the class back together for a whole-class discussion.

Guide for Facilitating the Whole Class Debrief and Discussion

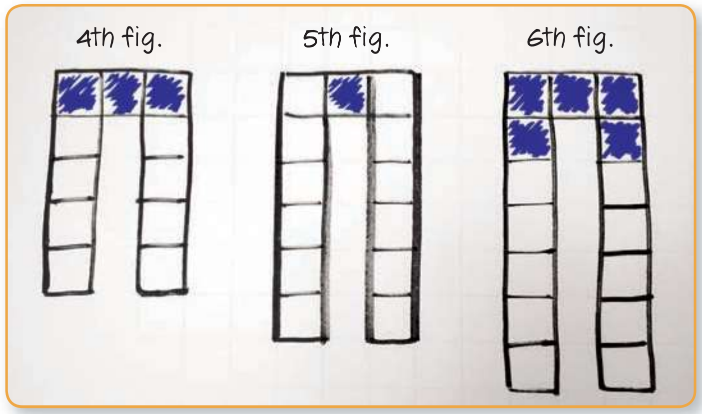
The goal in this scaffolded approach to visual patterns is to start concrete and get more and more abstract. If available, give students the option to use square-inch tiles to lay out the next two figures.

"Sketch the next two figures."

When going over this question, I tell students, *You are the brain and I am the hand.* From their desks I have them give me explicit instructions for how they want me to draw the 4th figure. There are a few different ways they might be looking at the figures and you want to draw out as many different ways of seeing the pattern as possible. What is important here is to get them to be clear about how they want you to start.

The first student might have you start by drawing the three squares across. Draw it the way they describe, next to the first three figures you have on the board. Color in the top three squares and then describe their way of seeing it—*So Roberta had me draw three across the top and then draw the legs underneath. Did anyone see it a different way?*

Try to find a student who drew the legs first and then added the square in the center of the top row (or, put another way, a student who drew the square in the center of the top row and then drew the legs coming down). Draw it and then color in the center square. Then ask if anyone saw it in a different way. If they did, you can have them describe how to draw the 6th figure.



When you are done, you'll have something like this *(see image at right)*:

"Complete the table."

Draw the table on the board and make sure you leave space below so you can continue the chart later in the debrief. Students will likely mention that they see a pattern in the number of squares: they go up by 2. If they don't bring it up, ask if anyone sees any patterns. Either way, write it on your table.

Ask the class how the table was helpful to their work. If it doesn't come up, mention that the table can help us organize the information without having to draw out all the figures. It also helps us identify more patterns.

This "going up by two (+2)" is the iterative (or recursive) rule. The iterative rule is a rule that you can use to find the value of a term (number of squares in this case) by using the value of the previous term. For example, if the 10th figure has 23 squares, I know the 11th figure has 2 more, or 25 squares.

"In a few sentences, describe what the tenth figure would look like."

Remember, you want a written description for this one. By all means, talk about the different ways students came up with 23 (and discuss any disagreement about that answer). But then get some descriptions on the board. Try to write down exactly what the student says. Then, draw a picture based on the description only—if you can playfully "misinterpret" any details and come up with a different figure, please do. Give students the opportunity to revise their description until your figure matches the picture they had in mind.

You should end up with descriptions similar to these:

> *"Draw three squares across. Then draw ten more squares down under the left square and another ten squares down from the right square."*

> *"Draw a column of 11 squares. Draw another square to the right of the top of the column. Draw another column of 11 squares down on the other side of the center square."*

You want at least one picture of the 10th figure on the board.

It's ok for the words to feel awkward. The eventual takeaway is that algebraic notation has a purpose. It is a tool that allows us to express our way of thinking clearly and concisely. Of course, you should not mention this yet.

"Explain how you would figure out the number of squares in the 99th figure."

TEACHER NOTE: Students can certainly use the iterative/recursive rule to answer this question and there is usually someone who continues the chart all the way to the 99th figure. Which is great. But this question also encourages students to look for an explicit rule. When we put students in a situation where they are telling themselves, "there has to be another way", they will often start looking for patterns. We should make this explicit whenever it happens.

An **explicit rule** is a rule that allows you to find the value of any term in the pattern without needing to know the value of the term before it. If you look at visual patterns in the context of functions, the explicit rule is the function rule. (And the **iterative/recursive rule** is the rate of change).

For this question, ask one of the groups that you identified during the group work problem-solving to come up and share their approach. After the group presentation(s), if it hasn't come up, ask the class how the 201 squares in the 99th figure would be arranged.

There are a few ways they might answer this one. They might recognize that if you draw three across the top, the number of squares down each column is equal to the figure number. They might also say, "We have 201 squares total. If we put three across the top, that leaves us with 198. Since both columns are equal, if we divide up the 198, we get 99 in each column."

For the other way of seeing the figure, they might recognize that after you put the one square in the center, the number of squares in each column is one more than the figure number. They may also use the 201 and say, "Well, after we put the 1 in the middle, we are left with 200 squares, so that is 100 on each side."

If they are not sure, you can refer to the two (or three) different ways of drawing the 10th figure and give them a few minutes to think about your question.

Teacher: How would *(student's name)* start drawing the 99th figure?

Student: She would draw three across the top.

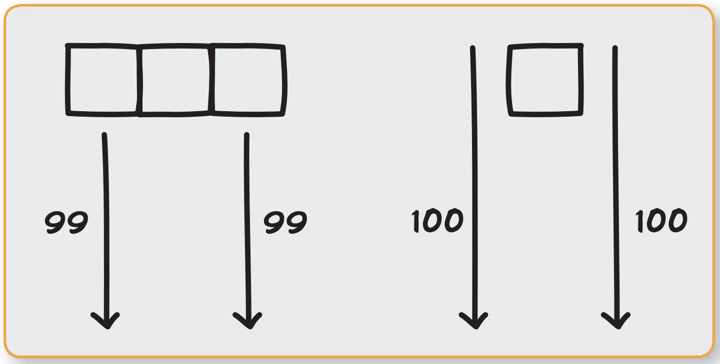
Teacher: How many squares would there be in each column going down?

Student: 99.

Teacher: What about *(student's name)*. How would she start drawing the 99th figure?

Student: She would draw the center square. She would then draw 100 squares down each side.

By the end, you want to create a visual sketch model that students can use as a shorthand.



"In a few sentences, describe how you would determine how many squares there are in any figure in the pattern."

Give the groups five minutes to work on the bonus question. Some of them who either didn't have time or were too intimidated to answer it before will have a little more confidence. If any groups finished it during the group problem-solving, ask them to try to use the debriefing notes from the board to try and find a second way.

After five minutes, bring the class back together. Remind them that when we look at information, make observations and find patterns, those patterns allow us make predictions.

The bonus question is asking us to predict how many squares there would be in any figure number, so let's look at how many squares there are in the figure numbers we already know.

Point to the visual sketch model you drew for the 99th figure. Ask how many squares are in the 99th figure. They'll either say *"99 + 99 + 3"* or *"2 × 99 + 3"*. Whichever form they use, record it on the board and use the same form for the rest.

Ask, *How did we arrange the 10th figure?*

And record their answer— either "10 + 10 + 3" or "2(10) + 3"

Continue all the way to the 1st figure.

Then, ask students how they could figure out the number of squares in the 47th figure, and record their answer: "47 + 47 + 3" or "2(47) + 3." Do it a few more times if it feels necessary.

Then ask your class: *What is changing? What is staying the same?*

Teacher: What is changing?

Students: The figure number.

Teacher: What is staying the same?

Students: We are always doubling the figure number and we are always adding three.

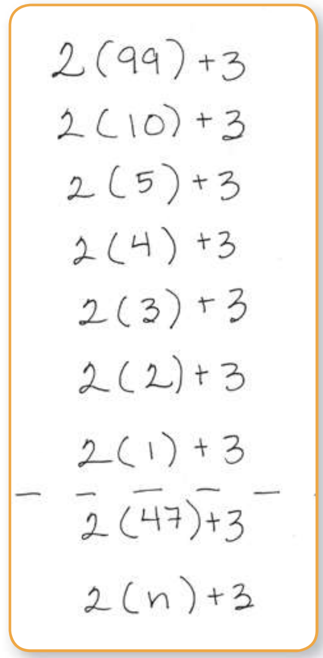
You want to start with the view of the figure that starts with the three squares across the top. When we get to generalizing the rule in a few steps, it is easier to work with the rule that has the simplest form of figure number in it. I recommend the first one, but the others can certainly be used as extensions for faster students. The first can be written as 99 + 99 + 3 and generalized as $2n + 3$. The other one is (99 + 1) + (99 + 1) +1 and generalized as $2(n + 1) + 1$. The rule that involves the 1st figure can be written as (99 – 1) + (99 – 1) + 5 and generalized as $2(n – 1) + 5$.

This is an opportunity to introduce students to writing 2 × 99 + 3 as 2(99) + 3.

If you prefer them to use the $2n + 3$ format, encourage them to do so by asking for another way to write adding a number to itself.

The first time you do visual patterns, focus on building only one of the ways of seeing the figure into an algebraic generalization. For future visual patterns, looking at more than one can be a great leaping off point for talking about equivalent equations.

Ask everyone to write in their notes, the steps they would take to find the number of squares in any figure number. Walk around and see what folks are writing and decide a few to share. Take a volunteer and record their steps on the board. Ask if their steps would always work. Have the class test it with the figure numbers we've already worked with, until everyone agrees it would work. Then ask if anyone has a different way.



Once you have at least one clear set of steps on the board that works, under the 47 + 47 + 3 or the 2(47) + 3, write an *n*. Say, *I don't know what figure number this is, but whatever figure number it is, what will I do to figure out the number of squares in it?*

Record their response—either *"n + n + 3"* or *2(n)+3.*

At the end, this piece of the board should look something like the image on the right:

9

Using a variable to stand for the independent change in a function—as we are using it here—is only one way to use a variable.

If you want to go a little deeper into this use of a variable, you might ask some follow-up questions: What does *n* represent? How many different numbers can *n* be?

9 **Ask if anyone knows the word for when we use letters like "*n*" to represent numbers.** Chances are, someone will throw out the word "variable." Ask them what it means when something varies and remind them of your question about what was changing and what was staying the same. It is the element that was changing—in this case, the figure number—that we represent with a variable. Have students add the following definition of a variable to their notes: *"When working with visual patterns, the variable is the part of an explicit rule that changes for each figure."*

10 **Give out The Arch Problem, part 3.** It could be either the final problem-solving activity of class or it could be given as a homefun assignment.

NOTE TO TEACHER: It's too much to get into this the first time you look at a visual pattern, but this question, "*Which figure will have ____ squares*?" can be used to build towards solving one-variable equations—i.e. solving for "*n*" in the equation 2*n* + 3 = 175. But hold off on making that explicit until after they've come up with their own methods and shared them. Give students a chance to work on it as it is written and it can become the foundation of students solving for a specific unknown, except instead of us having to tell students how to do it, they can tell us.

Debriefing this question is a good moment to compare an equation using a variable that has one specific value to a function, which is about the functional relationship, where the variable can be any number. Ask how this use of the variable is different from the way we used it when we came up with the rule, **$2n + 3$**.

- **"Which figure will have 175 squares in it?"**

There are a few strategies you should look for to have students share.

Some students might use guess and check. You should go over this method first.

They already know the 99th figure has 201 squares, so they know it is smaller than the 99th figure. Say, they start with the 80th figure. They will test each guess by using the rule—doubling the figure number and adding three—until they get to the 86th figure and realize the $2(86) + 3 = 175$.

Some might refer to the visual sketch of the figure and say something like, "Imagine you have 175 squares. You put three across the top, leaving you with 172 squares. You divide the 172 squares by 2 and divide them evenly to each side of the figure. That would give us a figure with three squares across and 86 squares down each side. So it is the 86th figure."

- **"Which figure will have 44 squares in it?"**

This is a bonus question because it is a trick question. There is no figure in this visual pattern that will have 44 squares in it. Many students will come to that conclusion, but the goal is for them to be able to explain why and how they know that is the case.

You need to be able to subtract 3 from the number and end up with an even number. Put another way, if you put the three squares across the top, you need an even number of squares to distribute between the two columns. 44 minus 3 is 41, which cannot be divided evenly into two columns.

11 **Re-write "Algebra is the generalization of arithmetic" on the board.** Ask students to do a pair share and discuss how the sentences connects to the work they've been doing on the arch problem.

This math log was adapted from *A Collection of Math Lessons: From Grades 6 through 8* by Marilyn Burns and Cathy Humphreys

Check-Out/Exit Ticket

- **Give students a minute or two to look over the whole board. Tell them you are going to give them some time to reflect on what they learned in class.**
- **On a separate piece of paper—you'll be collecting it—have students take notes on the following items:**
  - The date of the lesson.
  - What do you think would be a good title for today's class?
  - What happened in class today?
  - What did you learn about today?
  - What do patterns have to do with algebra?